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Antifield BRST quantization of duality-symmetric Maxwell theory

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Abstract: We perform the antifield BRST quantization of duality-symmetric Maxwell theory and show explicitly the quantum equivalence of the different formulations (covariant and non-covariant). The non-covariant gauge-fixed action is used in the computation of propagators for this model.

Keywords: BRST Quantization, Duality in Gauge Field Theories, BRST Symmetry.

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Contents

1. Introduction

In the past few years the concept of duality played a central role in field and string theory. Dualities became systematically studied in the literature once their importance in connecting apparently different string theories was realized. For instance S-duality establishes a correspondence between weak and strong coupled models and a special case of S-duality is represented by the electric-magnetic duality. Thus, the necessity of studying such a duality required a dual-symmetric action for the Maxwell theory. This manifest duality symmetry can be elegantly reformulated in terms of a self-duality condition on a complex field strength. Abelian p-forms, with $(p+1)$-field strengths satisfying a self-duality condition (Hodge duality), are only defined in $2(p+1)$ dimensions. Because of the minkowskian signature, the square of the Hodge dual $*$ is the identity in twice odd dimensions and minus the identity in twice even dimensions Thus, the condition: $F = *F$ allows non-trivial solutions

($F \neq 0$) for real fields only in twice odd dimensions: the chiral p-forms. In twice even dimensions, we have to take F to be a complex field and redefine the dual operator to be imaginary by $* \to \mathrm{i}*$. The complexification of the fields is also equivalent to the dualization of a pair of real p-forms gauge fields, like duality-symmetric Maxwell theory ($p = 1$).

The connection of chiral p-forms to supergravity [1] or branes and M-theory [2] was one of the motivations for a methodical approach of the subject. Several non-covariant actions [3, 4, 5] were proposed for the description of chiral p-forms. The main obstacle encountered in the construction of an action with manifest Lorentz-invariance was the presence of the self-duality requirement. Nevertheless, the problem was solved either by introduction of an infinite set of auxiliary fields entering the lagrangian in a polynomial way [6, 7] or using one auxiliary field in a non-polynomial way [8, 9]. Efforts for implementing the duality symmetry in Maxwell theory at the level of its action have been undertaken since the seventies [10, 11]. The topic has been addressed again over the last decade in a series of papers [12, 13]. This led to non-covariant versions [11, 12] or to lagrangians with manifest space-time symmetry [13, 14].

The quantization of theories containing chiral p-forms has been already performed for several values of p and different formulations of the systems. The covariant hamiltonian BRST (Becchi-Rouet-Stora-Tyutin) quantization of one chiral boson was realized in [6] and generalized to chiral p-forms in [7], applying the formulation of infinitely many ghosts. On the other hand, chiral 2–forms in 6 dimensions have been recently quantized [15] within the covariant BV (Batalin-Vilkovisky) treatment making use of various gauge-fixing conditions. The BV method has been also adopted [16] in proving the quantum equivalence of Schwarz-Sen [12] and Maxwell [11] theories. Nevertheless, the generating functionals derived in [16] do not exhibit a manifest Lorentz covariance. The aim of the present work is to obtain a correct path-integral for the covariant duality-symmetric Maxwell theory. As, in the first instance, we want to get a generating functional with manifest Lorentz symmetry we will base our considerations on the action proposed by Pasti, Sorokin and Tonin (PST) in [13]. The presence of the auxiliary field coupling (non-polynomially) to the two gauge potentials makes the gauge algebra non-Abelian, with field-dependent structure *functions*. As a consequence, we must choose a suitable quantization procedure. We will consider here the antifield-BRST method [19] because it proved, in the last twenty years, to be a very powerful quantization technique applicable also for models with open and/or non-Abelian (field-dependent) algebras, as it will be the case for us.

The paper is structured as follows. In section 2 we present the action and its gauge symmetries together with the gauge algebra. We compute then, in section 3, the minimal solution of the master equation and we infer also the BRST symmetry. By a well chosen non-minimal sector and an adequate gauge-fixing fermion the remaining gauge invariances will be fixed in section 4. The non-covariant gauge is the starting point in proving the quantum equivalence between the PST and the Maxwell theories as explained in section 5. It is afterwards used to explicitly determine the Feynman rules for the interaction of PST with gravity in section 6. In the last part, section 7, we collect and discuss our results.

2. Gauge symmetries of the classical action

We start our discussion by considering the PST action proposed to manifestly implement two symmetries in the description of free Maxwell theory: Lorentz invariance and electric-magnetic duality.

After fixing the notation, we emphasize the physical content of this model. Next, we briefly present its gauge algebra.

The PST action [13] constructed for the description of self-dual vector field is

$$S_0 = \int d^4x \left(-\frac{1}{8} F^{\alpha}_{mn} F^{\alpha mn} + \frac{1}{4(-u_l u^l)} u^m \mathcal{F}^{\alpha}_{mn} \mathcal{F}^{\alpha np} u_p \right) , \tag{2.1}$$

where the $m, n, \ldots$ stand for Lorentz indices in 4 dimensional space-time with a flat metric $(-,+,+,+)$. As explained in the introduction the Lagrangian contains two gauge potentials $(A^{\alpha}_m)_{\alpha=1,2}$ and one auxiliary field a, appearing here only as the gradient $u_m = \partial_m a$. The notation used throughout this paper is

$$\begin{aligned} u^2 &= u^m u_m \,, & v_m &= \frac{u_m}{\sqrt{-u^2}} \,, \\ F^{\alpha}_{mn} &= 2\partial_{[m} A^{\alpha}_{n]} \,, & F^{*\alpha}_{mn} &= \frac{1}{2}\epsilon_{mnpq} F^{\alpha pq} \,, \\ \mathcal{F}^{\alpha}_{mn} &= \mathcal{L}^{\alpha\beta} F^{\beta}_{mn} - F^{*\alpha}_{mn} \,, & H^{(-)\alpha}_m &= \mathcal{F}^{\alpha}_{mn} v^n \end{aligned} \tag{2.2}$$

with $\mathcal{L}^{\alpha\beta}$ being the antisymmetric unit matrix of SO(2). The equations of motion associated to (2.1) read

$$\delta A^{\alpha}_m : \qquad \epsilon^{mnqp} \partial_n (v_p H^{(-)\alpha}_q) = 0 \,, \tag{2.3}$$

$$\delta u_m : \qquad \frac{1}{2\sqrt{-u^2}} \left(H^{(-)\alpha}_n \mathcal{F}^{\alpha mn} - H^{(-)\alpha}_n H^{(-)\alpha n} v^m \right) = 0 \,. \tag{2.4}$$

It is straightforward to check the following gauge invariances of (2.1)

$$\delta_I A^{\alpha}_m = \partial_m \varphi^{\alpha} \,, \qquad \delta_I a = 0 \,, \tag{2.5}$$

$$\delta_{II} A^{\alpha}_m = -\mathcal{L}^{\alpha\beta} H^{(-)\beta}_m \frac{\phi}{\sqrt{-u^2}} \,, \qquad \delta_{II} a = \phi \,, \tag{2.6}$$

$$\delta_{III} A^{\alpha}_m = u_m \varepsilon^{\alpha} \,, \qquad \delta_{III} a = 0 \,, \tag{2.7}$$

that are irreducible. Pasti-Sorokin-Tonin have shown [13] that this model is in fact classically equivalent with Schwarz-Sen action [12] describing the dynamics of a single Maxwell field. Indeed, using the equations of motion (2.3) one can fix the gauge degrees of freedom of (2.7) in such a way that the self-duality condition

$$\mathcal{F}^{\alpha}_{mn} = 0 \tag{2.8}$$

is satisfied. Such a consequence of the equations of motion allows us to express one of the gauge fields A^{α}_{m} as function of the other one yielding the usual Maxwell Lagrangian (with remaining symmetry (2.5)) plus a contribution of u_m field. Further, one remarks from the second invariance (2.6) that a is pure gauge. Another way to see that is by expressing the field equation for a as a consequence of the equation of motion for A^{α}_{m}. That is why a can be easily fixed away using a clever gauge condition (avoiding the singularity $u^2 = 0$). So, the field u_m as well as one of the two A^{α}_{m} are auxiliary in the sence that one needs them only to lift self-duality and Lorentz invariance at the rank of manifest symmetries of the action. But, they can be removed on the mass-shell taking into account the gauge invariances of the new system. Nevertheless, the way we gauge fix the last invariance (2.7) can be applied only at the classical level since we make explicit use of the field equations, which cannot be done in a BRST path integral approach. The manner of fixing the unphysical degrees of freedom in the BRST formalism will be clarified in section 4.

Computing the gauge algebra we get

$$[\delta_{II}(\phi_1), \delta_{II}(\phi_2)] = \delta_{III}\left(\frac{\mathcal{L}^{\alpha\beta}H_p^{(-)\beta}}{(-u^2)^{3/2}}(\phi_1\partial^p\phi_2 - \phi_2\partial^p\phi_1)\right), \tag{2.9}$$

$$[\delta_{II}(\phi), \delta_{III}(\varepsilon^{\alpha})] = \delta_I(\phi\varepsilon^{\alpha}) + \delta_{III}\left(\frac{u^p\phi}{(-u^2)}\partial_p\varepsilon^{\alpha}\right). \tag{2.10}$$

Thus, our system describes a non-Abelian theory with the structure constants replaced by non-polynomial structure *functions*.

3. Minimal solution of the master equation

Having made the classical analysis of the model, we can start now the standard BRST procedure.[1] The first step is to construct the minimal solution of the master equation with the help of the gauge algebra. In order to reach that end we will introduce some new fields called ghosts and their antibracket conjugates known as antifields.

The minimal sector of fields and antifields dictated by the gauge invariances (2.5) –(2.7) as well as their ghost numbers and statistics are listed in table 1.

[1]For a short review of antifield BRST method see appendix A.

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Φ	A_m^α	a	$A_m^{\alpha *}$	a^*	c^α	c	c'^α	$c^{\alpha *}$	c^*	$c'^{\alpha *}$
$gh(\Phi)$	0	0	-1	-1	1	1	1	-2	-2	-2
antigh(Φ)	0	0	1	1	0	0	0	2	2	2
stat(Φ)	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$+$

Table 1: Ghost number, antighost number and statistics of the minimal fields and their antifields.

The transformations (2.5)–(2.7) determine directly the antigh number one piece of the extended action, i.e.

$$S_1 = \int d^4\, x \left[A_m^{\alpha *} \left(\partial^m c^\alpha - \mathcal{L}^{\alpha\beta} H_m^{(-)\beta} \frac{c}{\sqrt{-u^2}} + u^m c'^\alpha \right) + a^* c \right] . \tag{3.1}$$

In order to take into account the structure functions one has to insert in the solution of the master equation a contribution with antigh number two of the form

$$S_2 = \int d^4\, x \left[c'^{\alpha *} \left(\frac{\mathcal{L}^{\alpha\beta} H_p^{(-)\beta}}{(-u^2)^{3/2}} c\, \partial^p c + \frac{v_p}{\sqrt{-u^2}} c \partial^p c'^\alpha \right) + c^{\alpha *} c\, c'^\alpha \right] . \tag{3.2}$$

Due to the field dependence of the structure functions one should expect that S_1 and S_2 are not enough to completely determine the extended action and one will need an extra piece of antigh number three to do the job. Indeed, that was already the case for chiral 2–forms in 6 dimensions discussed in [15]. Nevertheless, one can readily check that in the present situation $S_{\rm min} = S_0 + S_1 + S_2$ is the minimal solution of the classical master equation $(S_{\rm min}, S_{\rm min}) = 0$, i.e.

$$\begin{aligned} (S_1, S_1)_1 + 2(S_1, S_1)_1 &= 0\, , \\ (S_2, S_2)_2 + 2(S_1, S_2)_2 &= 0\, . \end{aligned} \tag{3.3}$$

This follows also as a consequence of the irreducibility of our model. The way we constructed $S_{\rm min}$ will ensure also a properness condition which says that the rank of the hessian

$$S_{\tilde{C}}{}^{\tilde{A}} = \omega^{\tilde{A}\tilde{B}} \frac{\delta^L \delta^R S_{\rm min}}{\delta \Phi^{\tilde{B}} \delta \Phi^{\tilde{C}}} \tag{3.4}$$

at the stationary surface corresponds to precisely to half its dimension (where $(\Phi^{\tilde{A}})_{\tilde{A}=1,\dots,2N}$ labels all the fields and antifields in the minimal sector, while $\omega^{\tilde{A}\tilde{B}}$ denotes the symplectic matrix in $2N$ dimensions). Such a condition expresses that $S_{\rm min}$ has only a number of gauge invariances equal to N, not $2N$ as one could superficially think.

Once $S_{\rm min}$ has been derived, we can infer the BRST operator s, which is the sum of three operator of different antigh number

$$s = \delta + \gamma + \rho\, . \tag{3.5}$$

For instance, the non-trivial action of the Koszul-Tate differential, of antigh number -1, is in our case given by

$$\delta A^{\alpha *}_m = \epsilon^{mnqp}\partial_n\big(v_p H^{(-)\alpha}_q\big)\,, \tag{3.6}$$

$$\delta a^* = \partial_m\left(\frac{1}{2\sqrt{-u^2}}\left(H^{(-)\alpha}_n\mathcal{F}^{\alpha mn} - H^{(-)\alpha}_n H^{(-)\alpha n}v^m\right)\right)\,, \tag{3.7}$$

$$\delta c^{\alpha *} = -\partial^m A^{\alpha *}_m\,, \tag{3.8}$$

$$\delta c^* = -\frac{\mathcal{L}^{\alpha\beta}H^{(-)\beta p}}{\sqrt{-u^2}}A^{\alpha *}_p + a^*\,, \tag{3.9}$$

$$\delta c'^{\alpha *} = u^m A^{\alpha *}_m\,. \tag{3.10}$$

The third piece, ρ, of antigh number $+1$ is present also because the structure functions determined by (2.5)-(2.7) depend explicitly on the fields.

In this way the goal of this section, i.e. the construction of the minimal solution for the master equation, was achieved.

4. The gauge-fixed action

The minimal solution $S_{\rm min}$ will not suffice in fixing all the gauge invariances of the system and, before fixing the gauge, one needs a non-minimal solution for $(S, S) = 0$ in order to take into account the trivial gauge transformations. In this section we first construct such a non-minimal solution and, afterwards, we propose two possible gauge-fixing conditions which will yield two versions for the gauged-fixed action: a covariant and a non-covariant one.

4.1 Non-minimal sector

Inspired by the gauge transformations (2.5)-(2.7) and their irreducibility we propose a non-minimal sector given in table 2.

They satisfy the following equations

$$\begin{aligned} s\bar{C}^{\cdots} &= B^{\cdots}\,,\\ sB^{\cdots} &= 0\,,\\ sB^{\cdots *} &= \bar{C}^{\cdots *}\,,\\ s\bar{C}^{\cdots *} &= 0\,. \end{aligned} \tag{4.1}$$

Φ	B^α	B	B'^α	$\bar{C}^\alpha$	$\bar{C}$	$\bar{C}'^\alpha$	$B^{\alpha *}$	B^*	$B'^{\alpha *}$	$\bar{C}^{\alpha *}$	$\bar{C}^*$	$\bar{C}'^{\alpha *}$
$gh(\Phi)$	0	0	0	-1	-1	-1	-1	-1	-1	0	0	0
$\mathrm{stat}(\Phi)$	+	+	+	$-$	$-$	$-$	$-$	$-$	$-$	+	+	+

Table 2: Ghost number and statistics of the non-minimal fields and their antifields.

The dots are there to express that these relations are valid for the correspondingly three kinds of non-minimal fields. We immediately see that $\bar{C}^{\cdots}$'s and $B^{\cdots}$'s constitute trivial pairs, as well as their respective antifields, in such a way that they do not enter in the cohomology of s. Hence, they are called non-minimal. A satisfactory explanation of the necessity of the presence of a non-minimal sector is provided by BRST-anti-BRST formalism. Their contribution to the solution of the master equation is

$$S_{\mathrm{non-min}} = S_{\mathrm{min}} + \int d^4\, x \left(\bar{C}^{\alpha *} B^\alpha + \bar{C}^* B + \bar{C}'^{\alpha *} B'^\alpha \right) . \tag{4.2}$$

4.2 Covariant gauge fixing

We will first try a covariant gauge fixing that in principle should yield a covariant gauge-fixed action and we will see what is the main problem that occurs. One can consider the following *covariant* gauge choices

$$\delta_I \to \partial^m A^\alpha_m = 0 \, , \tag{4.3}$$

$$\delta_{II} \to u^2 + 1 = 0 \, , \tag{4.4}$$

$$\delta_{III} \to u^m A^\alpha_m = 0 \, . \tag{4.5}$$

The gauge choice (4.3) is analogous to the Lorentz gauge. In its turn (4.4) allows to take a particular Lorentz frame in which $u^m(x)$ is the unit time vector at the point x. In such a case, at the point x, (4.5) is the temporal gauge condition for the two potentials.

A gauge-fixing fermion corresponding to the gauge choices (4.3)-(4.5) is

$$\Psi[\Phi^A] = - \int d^4\, x \left[\bar{C}^\alpha \partial^m A^\alpha_m + \bar{C}(u^2 + 1) + \bar{C}'^\alpha u^m A^\alpha_m \right] . \tag{4.6}$$

One expresses now all the antifields with the help of $\Psi[\Phi]$, i.e.

$$\Phi^*_A = \frac{\delta \Psi[\Phi^A]}{\delta \Phi^A} \tag{4.7}$$

getting

$$\begin{aligned}
&A^{\alpha *}_m = \partial_m \bar{C}^\alpha - u_m \bar{C}'^\alpha \, , \qquad a^* = 2\partial_m (u^m \bar{C}) + \partial^m (A^\alpha_m \bar{C}'^\alpha) \, , \\
&c^{\cdots *} = 0 \, , \qquad B^{\cdots *} = 0 \, , \\
&\bar{C}^{\alpha *} = -\partial^m A^\alpha_m \, , \qquad \bar{C}^* = -(u^2+1) \qquad \bar{C}'^{\alpha *} = -u^m A^\alpha_m \, .
\end{aligned}$$

Using the last relations one can find the gauge fixed action as in (A.11) which in our case reads

$$\begin{aligned}
S_\Psi = S_0 + \int d^4\, x \Big[& -\bar{C}^\alpha \Box c^\alpha - \frac{\mathcal{L}^{\alpha\beta} H^{(-)\beta m}}{\sqrt{-u^2}} \partial_m \bar{C}^\alpha \cdot c + u^m \partial_m \bar{C}^\alpha \cdot c'^\alpha - \\
& - u_m \bar{C}'^\alpha \partial^m c^\alpha - u^2 \bar{C}'^\alpha c'^\alpha - (2 u_m \bar{C} + A^\alpha_m \bar{C}'^\alpha) \partial^m c - \\
& - (\partial^m A^\alpha_m) B^\alpha - (u^2+1) B - (u^m A^\alpha_m) B'^\alpha \Big] .
\end{aligned} \tag{4.8}$$

Writing down the path integral (A.12), one can integrate directly the fields $B^{\cdots}$ producing the gauge conditions (4.3)-(4.5). A further integration of $\bar{C}$, $\bar{C}'^{\alpha}$ and c'^{α} (in this order) leads to

$$Z = \int \left[\mathcal{D}A\mathcal{D}a\mathcal{D}c^{\alpha}\mathcal{D}c\mathcal{D}\bar{C}^{\alpha}\right]\, \delta(\partial^m A^{\alpha}_m)\, \delta(u^2+1)\, \delta(u^m A^{\alpha}_m)\, \delta(u^m\partial_m c) \exp iS'_{\Psi}\,, \quad (4.9)$$

where

$$S'_{\Psi} = \int d^4x \left[-\bar{C}^{\alpha}\Box c^{\alpha} + \left(\frac{\mathcal{L}^{\alpha\beta}H^{(-)\beta m}}{\sqrt{-u^2}}c + \frac{u^m}{u^2}(u^p\partial_p c^{\alpha} + A^{\alpha}_p\partial^p c)\right)\partial_m\bar{C}^{\alpha}\right]. \quad (4.10)$$

Of course, the next step in getting a covariant generating functional from which we should read out the *covariant* propagator for the fields A^{α}_m would be the elimination of c and a in (4.9). Due to the "gauge condition" for the ghost c (i.e. $u^m\partial_m c = 0$) and the way it enters the gauge-fixed action S'_{Ψ}, this integration is technically difficult. What one could try is to integrate both c and a at the same time. This is also not straightforwardly possible as a consequence of the gauge condition (4.4). This requirement was necessary to *covariantly* fix the symmetry (2.6). Nevertheless, one can attempt to find the general solution to this equation (4.4), which reduces to the integration of $\partial^m a = \Lambda^m{}_p(x)n^p$ (with $\Lambda^m{}_p(x)$ a point-dependent Lorentz boost and n_p a constant time-like vector, i.e. $n_p n^p = -1$). Such a solution is still unconvenient due to x-dependence of the Lorentz transformation matrix $\Lambda^m{}_p(x)$.

A way to overcome this sort of complication is to choose a particular form for this matrix, breaking Lorentz symmetry. It is precisely this price that we have to pay in order to explicitly derive the propagator of A^{α}_m fields. As it will be explained in the next subsection, by taking a particular solution for (4.4), i.e. by giving up Lorentz invariance, we will be able to express the gauged-fixed action in a more convenient form for our purposes.

4.3 Non-covariant gauge fixing

As it was remarked in the previous subsection in order to explicitly derive the Feynman rules for the PST model one has to break up its Lorentz symmetry by taking a specific solution of the equation (4.4). In this subsection we present a non-covariant gauge of the theory and the advantages for such a choice will become clear in the next sections. A possible *non-covariant* gauge fixing is

$$\delta_I \to \partial^m A^{\alpha}_m = 0\,, \quad (4.11)$$

$$\delta_{II} \to a - n_m x^m = 0\,, \qquad n_m n^m = -1\,, \quad (4.12)$$

$$\delta_{III} \to n^m A^{\alpha}_m = 0\,. \quad (4.13)$$

By (4.12), the gradient $\partial^m a$ becomes equal to the vector n^m introduced above. In a Lorentz frame where $n^m = (1,0,0,0)$ the requirement (4.13) is the temporal gauge condition and (4.11) the Coulomb gauge condition for the two potentials A^{α}_m.

Then, the gauge-fixing fermion will be

$$\Psi[\Phi^A] = -\int d^4\,x\,\left[\bar{C}^\alpha \partial^m A_m^\alpha + \bar{C}(a - n_m x^m) + \bar{C}'^\alpha n^m A_m^\alpha\right]. \tag{4.14}$$

Using the same non-minimal contribution $S_{non-min}$ as before, the non-covariant gauge-fixed action is

$$\begin{aligned} S_\Psi = S_0 + \int d^4\,x \Bigg[& \left(\partial_m \bar{C}^\alpha - u_m \bar{C}'^\alpha\right)\left(\partial^m c^\alpha - \mathcal{L}^{\alpha\beta} H_m^{(-)\beta} \frac{c}{\sqrt{-u^2}} + u^m c'^\alpha\right) + \\ & + \left(-\bar{C} + \partial^m (A_m^\alpha \bar{C}'^\alpha)\right) c - \partial^m A_m^\alpha B^\alpha - (a - n_m x^m) B - \\ & - u^m A_m^\alpha B'^\alpha \Bigg]. \end{aligned} \tag{4.15}$$

This action is by far more convenient in deriving the propagator of the gauge fields than its covariant expression (4.10) because one can completely integrate the ghost sector. Also, the bosonic part takes a more familiar form. The quantum equivalence of the PST model with ordinary Maxwell theory will be based also on this non-covariant action.

5. Path integral, quantum equivalence of PST action with Maxwell theory

The gauge-fixed action corresponding to the non-covariant gauge choice can be used to recover the Schwarz-Sen theory, which is itself equivalent to the Maxwell theory. The generating functional is taken to be (see appendix A and B)

$$Z = \int \mathcal{D}A_m^\alpha\, \mathcal{D}a\, \mathcal{D}c^{\cdots}\, \mathcal{D}B^{\cdots}\, \mathcal{D}\bar{C}^{\cdots} \det(\Box) \det{}^{-1}(\mathrm{curl})\, \exp iS_\Psi\,, \tag{5.1}$$

where S_Ψ is given by (4.15).

After integrating out some fields, in the following order ($B^{\cdots}$, $\bar{C}$, c, $\bar{C}'^\alpha$, c'^α, a), we obtain the path integral

$$Z = \int \mathcal{D}A_m^\alpha\, \mathcal{D}\bar{C}^\alpha\, \mathcal{D}c^\alpha \det(\Box)\, \det{}^{-1}(\mathrm{curl})\, \delta(\partial^m A_m^\alpha)\, \delta(n^m A_m^\alpha)\, \exp iS'_\Psi\,, \tag{5.2}$$

where the gauge-fixed action reduces now to

$$S'_\Psi = \int d^4\,x \left[-\frac{1}{2} n^m F_{mn}^{*\alpha} \mathcal{F}^{\alpha n p} n_p - \bar{C}^\alpha \Box c^\alpha - \bar{C}'^\alpha n^p n^q \partial_p \partial_q c^\alpha\right]. \tag{5.3}$$

If we place ourselves in a Lorentz frame where $n^m = (1,0,0,0)$, the functional S'_Ψ assumes the form of the sum of Schwarz-Sen gauge-fixed action (B.13) and a ghost term

$$-\int d^4x\, \bar{C}^\alpha \triangle c^\alpha\,. \tag{5.4}$$

At this point we can integrate the ghosts $\bar{C}^\alpha$ and c^α, and the two fields A_0^α, obtaining exactly the generating functional (B.12) of the Maxwell theory in the non-covariant formulation (The quantum equivalence of Maxwell and Schwarz-Sen actions is briefly reviewed in appendix B).

This proves the (quantum) equivalence of the PST action (2.1) with the Maxwell theory, which was already known at the classical level. The quantum equivalence was not obvious because the PST action of the free Maxwell theory is not quadratic (and so the path integral is not gaussian) and the pure gauge field a is not, strictly speaking, an auxiliary field (its equation of motion is not an *algebraic* relation which allows its elimination from the action).

As a last remark, we notice that the bosonic part of the action (5.3) produces two poles in the propagator of the gauge fields A_i^α : one physical, the usual $1/\Box$, the other is an *apparent* unphysical pole of type $-1/(\partial_N^2+\Box)$ (i.e. the inverse of laplacian operator in an appropriate Lorentz frame, as $\partial_N \equiv n^p\partial_p$), also present for the Schwarz-Sen theory. This is not a physical pole because it corresponds to modes that do not propagate. This pole appears also in another non-covariant gauge choice: the Coulomb gauge in Maxwell theory. We will try to clarify this point in the next section using the example of PST model coupled to gravity. In that case, we can see explicitly that the unphysical mode do not contribute at all to scattering amplitudes.

6. Coupling to gravity

The goal of this section is to show that the massless propagator $1/\Box$, rather than $-1/(\partial_N^2+\Box)$, contributes to the Feynman diagrams of the vector fields A_m^α for the particular case of PST theory coupled to gravity.

We consider now the same PST action (2.1) but in a gravitational background characterized by a metric $g_{\mu\nu}$, i.e. we take[2]

$$S_0^g = -\frac{1}{2}\int \sqrt{-g}\, v_\mu H_\nu^{(-)\alpha} g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma}^{*\alpha}\,, \tag{6.1}$$

where $g = \det g_{\mu\nu}$.

Next, we apply the same BRST formalism as before, following precisely the same steps (just replacing the flat indices by curved ones). Moreover, using the same non-covariant gauge, one infers for the ghost sector a similar contribution of type (5.4), which becomes here

$$-\int d^4 x\sqrt{-g}\,\bar{C}^\alpha(\Box_{cov}+\nabla_N^{cov}\nabla_N^{cov})c^\alpha\,. \tag{6.2}$$

The only difference resides in replacing the ordinary derivatives by covariant quantities. In any case, the fermionic ghosts decouple from the bosonic fields and can

[2]The greek letters μ, ν, ρ etc. label curved indices, while α and β denote SO(2) indices.

be handled as explained after (5.4) (by integrating over them in the path integral). This is the reason way we focus our attention on the bosonic part of the gauge-fixed action arising from the original action S_0^g.

As we are looking only for the first-order interaction of the model with the background $g_{\mu\nu}$ it is natural to try to expand this metric around the flat one. In other words, we consider

$$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu} \tag{6.3}$$

and, for further convenience, we assume that the fluctuation $h^{\mu\nu}$ can be parametrized in terms of inverse e^μ_m of the orthogonal vectors e^m_μ, i.e.

$$h^{\mu\nu} = e^\mu_m e^\nu_n \eta^{mn} \tag{6.4}$$

(for simplicity the label m will be suppressed in the future considerations).

Our next move consists in developing the bosonic part of the gauge-fixed action to the first-order in the perturbation $h^{\mu\nu}$. [3] After some computation one gets

$$S^g_\Psi = \int d^4\,x\Big\{-\frac{1}{2}A^\alpha_\mu[\delta^{\alpha\beta}\eta^{\mu\nu}(-\Box - \partial^2_N) + \mathcal{L}^{\alpha\beta}T^{\mu\nu}\partial_N]A^\beta_\nu + \tag{6.5}$$

$$+\frac{1}{2}(T^{\mu\nu}A^\alpha_\nu)\Big[\delta^{\alpha\beta}\eta_{\mu\sigma}(\frac{1}{2}\tilde{h} - (n^\tau e_\tau)^2) + \delta^{\alpha\beta}e_\mu e_\sigma - \tag{6.6}$$

$$-\frac{1}{2}\mathcal{L}^{\alpha\beta}(n^\zeta e_\zeta)\epsilon_{\mu\kappa\tau\sigma}e^\kappa n^\tau\Big](T^{\sigma\rho}A^\beta_\rho)\Big\}\,, \tag{6.7}$$

where we neglected the second order in $h^{\mu\nu}$ or higher. In the meantime we have employed the notation $\tilde{h} = h^{\mu\nu}\eta_{\mu\nu}$ and

$$T^{\mu\nu}A^\alpha_\nu = \epsilon^{\mu\nu\rho\sigma}n_\rho\partial_\sigma A^\alpha_\nu\,. \tag{6.8}$$

The object $T^{\mu\nu}$, defined in this way, is a differential operator transforming one-forms into one-forms. It is antisymmetric under the interchange of its indices and it is characterized by a very important feature, namely

$$T^{\mu\rho}T_{\rho\sigma}T^{\sigma\nu} = -(\Box + \partial^2_N)T^{\mu\nu}\,. \tag{6.9}$$

This property allows one to transform any series expansion in $T^{\mu\nu}$ into a polynomial containing only 1, T and T^2.

Let us return to the interpretation of the expansion (6.5)-(6.7). The first remark is that the zeroth-order, (6.5), coincides with the one from the flat space discussion.

This term delivers the gauge-fixed kinetic operator

$$K_{\mu\nu}{}^{\alpha\beta} = \delta^{\alpha\beta}\eta_{\mu\nu}(-\Box - \partial^2_N) + \mathcal{L}^{\alpha\beta}T_{\mu\nu}\partial_N\,, \tag{6.10}$$

[3]The indices are from now on raised and lowered with the flat metric $\eta^{\mu\nu}$.

whose inverse is nothing but the propagator ${P_{\mu\nu}}^{\alpha\beta}$ of the vector fields A^{α}_{μ}. Then a simple computation based on the property (6.9) of $T^{\mu\nu}$ gives the explicit form of the propagator

$$ {P_{\mu\nu}}^{\alpha\beta} = -\frac{1}{\Box + \partial_N^2}\left[\delta_{\mu\nu}\delta^{\alpha\beta} + \frac{\mathcal{L}^{\alpha\beta}T_{\mu\nu}\partial_N}{\Box} - \frac{\delta^{\alpha\beta}T_{\mu\rho}{T^{\rho}}_{\nu}\partial_N^2}{\Box(\Box + \partial_N^2)}\right]. \tag{6.11} $$

If we consider also the first-order interaction (6.6)-(6.7) with a gravitational background we notice that in such an interaction the gauge fields A^{α}_{μ} couple to the perturbation $h^{\mu\nu}$ only as $T^{\mu\nu}A^{\alpha}_{\nu}$. Therefore, we conclude that the effective propagator in the presence of gravity must be

$$ T^{\mu\rho}{P_{\rho\sigma}}^{\alpha\beta}T^{\sigma\nu} = [\mathcal{L}^{\alpha\beta}{\delta^{\mu}}_{\sigma}\partial_N - \delta^{\alpha\beta}{T^{\mu}}_{\sigma}]T^{\sigma\nu}\frac{1}{\Box}, \tag{6.12} $$

where we see that the apparent pole $-1/(\partial_N^2 + \Box)$ has been replaced by an expected massless propagator $1/\Box$. This should not be understood as a result of the specific gravitational coupling, but as a characteristic of Feynman computations for the PST model.

The expression of the effective propagator together with the interaction terms in S^{g}_{Ψ} can further be used to determine the building blocks of the one-loop Feynman diagrams for the coupling of the PST model to a gravitational background. A similar method was carried out in [17] in computing the gravitational anomalies in $4n+2$ dimensions.

7. Conclusions

In the present paper we demonstrated the equivalence of the PST and Schwarz-Sen formulations of duality-symmetric Maxwell theory at the quantum level. The latter, Schwarz-Sen, is quantum mechanically also equivalent to the ordinary Maxwell theory [16] such that all these models are physically related (on-shell) at the classical and quantum level, even if their off-shell descriptions are different.

To this end (to prove the equivalence) we have adopted the antifield-BRST quantization method. This approach resides in compensating all the gauge symmetries of the original system by some fermionic ghosts and their antibracket conjugates - called antifields. After extending the action to a suitable chosen non-minimal sector, we had to fix the gauge. We were able to perform two different gauge-fixings. The covariant one preserves Lorentz invariance but it has the disadvantage of an intricate form in the ghost sector which makes its integration difficult. On the other hand, giving up Lorentz symmetry we presented also a non-covariant gauge which has a simple structure in its fermionic part leading us to favourable results. Firstly, it was the cornerstone in proving the quantum equivalence of the studied PST model and the Schwarz-Sen action. Secondly, the correct Feynman rules have been infered. We used the example of gravitational interaction of the PST system in the same gauge condition to show explicitly that the unphysical pole of the propagator is a gauge artifact, of the same kind that the one appearing for usual Maxwell theory in Coulomb gauge. Such a first-order expansion in the perturbed metric was performed also by Lechner [17] in studying the gravitational anomalies of the self-dual tensors in $4n+2$ dimensions. However, as discussed in [18], the self-dual vector field in 4 dimensions should be anomaly free.

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A. Basic ingredients of antifield-BRST formalism

Here we give only some of the main ideas underlying the lagrangian BRST method. For more details we refer the reader to [19, 20].

Let $S_0[\phi^i]$ be an action with the following bosonic gauge transformations[4]

$$\delta_\varepsilon \phi^i = R^i_\alpha \epsilon^\alpha \tag{A.1}$$

which are irreducible. Then, one has to enlarge the "field" content to

$$\{\Phi^A\} = \{\phi^i, C^\alpha\}\,. \tag{A.2}$$

The fermionic ghosts C^α correspond to the parameters ε^α of the gauge transformations (A.1). To each field Φ^A we associate an antifield Φ^*_A of opposite parity. The set of associated antifields is then

$$\{\Phi^*_A\} = \{\phi^*_i, C^*_\alpha\}\,. \tag{A.3}$$

The fields possess a vanishing antighost number (antigh) and a nonvanishing pureghost number (pgh)

$$\mathrm{pgh}(\phi^i) = 0, \qquad \mathrm{pgh}(C^\alpha) = 1. \tag{A.4}$$

The pgh number of the antifields vanish but their respective antigh number is equal to

$$\mathrm{antigh}(\Phi^*_A) = 1 + \mathrm{pgh}(\Phi^A). \tag{A.5}$$

[4]We use the DeWitt notation.

The total ghost number (gh) equals the difference between the pgh number and the antigh number. The antibracket of two functionals $X[\Phi^A, \Phi^*_A]$ and $Y[\Phi^A, \Phi^*_A]$ is defined as

$$(X,Y) = \int d^n x \left(\frac{\delta^R X}{\delta \Phi^A(x)} \frac{\delta^L Y}{\delta \Phi^*_A(x)} - \frac{\delta^R X}{\delta \Phi^*_A(x)} \frac{\delta^L Y}{\delta \Phi^A(x)} \right) , \tag{A.6}$$

where $\delta^R/\delta Z(x)$ and $\delta^L/\delta Z(x)$ denote functional right- and left-derivatives.

The *extended action* S is defined by adding to the classical action S_0 terms containing the antifields in such a way that the classical *master equation*,

$$(S,S) = 0\,, \tag{A.7}$$

is satisfied, with the following boundary condition:

$$S = S_0 + \phi^*_i R^i_\alpha C^\alpha + \ldots \tag{A.8}$$

This imposes the value of terms quadratic in ghosts and antifields. The extended action has also to be of vanishing gh number. If the algebra is non-abelian, we know that we have to add other pieces of antigh number two in the extended action with the general form (due to structure functions)

$$S_2^{2a} = \frac{1}{2} C^*_\alpha f^\alpha_{\beta\gamma} C^\beta C^\gamma\,. \tag{A.9}$$

If the algebra is open, other terms in antigh number must be added, quadratic in ϕ^*_i's. Furthermore, other terms in higher antigh number could be necessary, e.g. when the structure functions depend on the fields ϕ^i.

The extended action captures all the information about the gauge structure of the theory: the Nöether identities, the (on-shell) closure of the gauge transformations and the higher order gauge identities are contained in the master equation.

The BRST transformation s in the antifield formalism is a canonical transformation, i.e. $sA = (A,S)$. It is a differential: $s^2 = 0$, its nilpotency being equivalent to the master equation (A.7). The BRST differential decomposes according to the antigh number as

$$s = \delta + \gamma + \text{"more"}$$

and provides the gauge invariant functions on the stationary surface, through its cohomology group at gh number zero $H_0(s)$. The Koszul-Tate differential δ

$$\delta \Phi^*_i = (\Phi^*_i, S)|_{\Phi^*_A = 0} \tag{A.10}$$

implements the restriction on the stationary surface, and the exterior derivative along the gauge orbits γ

$$\gamma \Phi^i = (\Phi^i, S)|_{\Phi^*_A = 0}$$

picks out the gauge-invariant functions.

The solution S of the master equation possesses gauge invariance, and thus, cannot be used directly in a path integral. There is one gauge symmetry for each field-antifield pair. The standard procedure to get rid of these gauge degrees of freedom is to use the *gauged-fixed action* S_Ψ defined by

$$S_\Psi = S_{\mathrm{non-min}}\left[\Phi^A, \Phi^*_A = \frac{\delta\Psi[\Phi^A]}{\delta\Phi^A}\right]. \tag{A.11}$$

The functional $\Psi[\Phi^A]$ is known as the *gauge-fixing fermion* and must be such that $S_\Psi[\Phi]$ is non-degenerate, i.e. the equations of motion derived from the gauge-fixed action $\delta S_\Psi[\Phi^A]/\delta\Phi^A = 0$ have unique solution for arbitrary initial conditions, which means that all gauge degrees of freedom have been eliminated. It also has to be local in order that the antifields are given by local functions of the fields.

The generating functional of the theory is

$$Z = \int [\mathcal{D}\Phi^A] \exp iS_\Psi\,. \tag{A.12}$$

The value of the path integral is independent of the choice of the gauge-fixing fermion Ψ. The notation $[\mathcal{D}\Phi]$ stands for $\mathcal{D}\Phi\,\mu[\Phi]$, where $\mu[\Phi]$ is the measure of the path integral. It is important to notice that the expression of the measure $\mu[\Phi]$ in this path integral is not completely determined by the Lagrangian approach. A correct way to determine it, would be to start from the Hamiltonian approach for which the choice of measure is trivial, indeed it is known to be $\mathcal{D}\Phi\mathcal{D}\Pi$, that is the product over time of the Liouville measure $d\Phi^A d\Pi_A$.

It can be proved that, if correctly handled, the two approaches are equivalent (see [20] and references therein). This justifies a posteriori the choice of the measure $\mu[\Phi]$ in (A.12).

B. Gauge-fixing of Maxwell theory

The generating functional for the Maxwell theory in the Hamiltonian approach is well known (see e.g. [16, 20])

$$Z = \int \mathcal{D}A_m\,\mathcal{D}\pi_m\,\mathcal{D}c\,\mathcal{D}\bar{\mathcal{P}}\,\mathcal{D}\bar{c}\,\mathcal{D}\mathcal{P}\ \exp iS^M_\Psi\,. \tag{B.1}$$

As usual, π_m is the conjugate momentum of A_m, c and $\bar{c}$ are ghosts, and $\bar{\mathcal{P}}$ and $\mathcal{P}$ are their respective conjugate momenta. The Hamiltonian gauge-fixed action in the Coulomb gauge is given by

$$S^M_\Psi = \int d^4x(\pi_m\dot{A}^m + \dot{c}\bar{\mathcal{P}} - \mathcal{H}_0 + A_0\partial_i\pi^i + \pi_0\partial^i A_i + i\bar{\mathcal{P}}\mathcal{P} - i\bar{c}\Box c)\,, \tag{B.2}$$

where i, j, ... stand for spatial indices in the 3 dimensional hyperplane x^0 constant. The Hamiltonian density is equal to

$$\mathcal{H}_0 = \frac{1}{2}(\pi^i \pi_i + B^i B_i)\,. \tag{B.3}$$

The magnetic field is $B^i = F^*_{0i}$. We can easily integrate the fields A^0, π_0, the ghosts c, $\bar{c}$ as well as their conjugate momenta $\mathcal{P}$, $\bar{\mathcal{P}}$. Then, we obtain that

$$Z = \int \mathcal{D}A_i\, \mathcal{D}\pi_i \det(\Box)\, \delta(\partial_i \pi^i)\, \delta(\partial^i A_i)\, \exp i\tilde{S}^M_\Psi \tag{B.4}$$

with

$$\tilde{S}^M_\Psi = \int d^4x(\pi_i \dot{A}^i - \mathcal{H}_0)\,. \tag{B.5}$$

The determinant of $\Box$ comes from the integration on the fermionic ghosts. The integration on A^0 and π_0 gives the delta-functions enforcing, respectively, the Gauss law and the Coulomb gauge.

In order to make the connection with the gauge-fixed Schwarz-Sen action, we have to move to a two-potential formulation, that is we have to solve the Gauss constraint $\partial_i \pi^i = 0$ by introducing a potential Z^i such that

$$\pi^i = \epsilon^{ijk}\partial_j Z_k\,. \tag{B.6}$$

The potential Z_i can be decomposed into a sum of a longitudinal and a transverse part: $Z_i = Z^L_i + Z^T_i$. When Z_i is transverse ($Z_i = Z^T_i$), the equation (B.6) is invertible (with appropriate boundary conditions). More precisely, in that case one expresses Z_i as

$$Z_i = -\,\triangle^{-1}\,\epsilon_{ijk}\partial^j \pi^k\,. \tag{B.7}$$

We can introduce the field Z^i in the path integral in the following way

$$Z = \int \mathcal{D}A_i\, \mathcal{D}\pi_i\, \mathcal{D}Z_i \det(\Box)\, \delta(\partial_i \pi^i)\, \delta(\partial^i A_i)\, \delta(Z^i + \triangle^{-1}\epsilon^{ijk}\partial_j \pi_k)\, \exp i\tilde{S}^M_\Psi\,. \tag{B.8}$$

In order to make the comparison with the Schwarz-Sen approach we will use the relation

$$\delta(Z^i + \triangle^{-1}\epsilon^{ijk}\partial_j \pi_k) = \delta(Z^{L\,i})\delta(Z^{T\,i} + \triangle^{-1}\epsilon^{ijk}\partial_j \pi_k) \tag{B.9}$$

with $\delta(Z^{L\,i}) = \delta(\partial_i Z^i)$. We also notice that

$$\delta(Z^{T\,i} + \triangle^{-1}\epsilon^{ijk}\partial_j \pi^T_k) \;=\; \underbrace{\det{}^{-1}(\triangle^{-1}\mathrm{curl})}_{=\det(\mathrm{curl})}\, \delta(\pi^{T\,i} - \epsilon^{ijk}\partial_j Z^T_k)\,, \tag{B.10}$$

where "curl" stands for the operator $\epsilon^{ijk}\partial_j$, and $\partial_i \pi^{T\,i} = 0$. We finally identify the two potentials as follows

$$A^1_i = A_i\,, \qquad A^2_i = Z_i\,. \tag{B.11}$$

Putting all these remarks together we can integrate out the π_i to obtain

$$Z = \int \mathcal{D}A_i^\alpha \det(\Box)\det(\mathrm{curl})\,\delta(\partial^i A_i^\alpha)\,\exp iS_\Psi^{S-S}\,, \tag{B.12}$$

where S_Ψ^{S-S} is the Schwarz-Sen gauge-fixed action

$$S_\Psi^{S-S} = \int d^4x\,\frac{1}{2}(\mathcal{L}^{\alpha\beta}\dot{A}_i^\alpha - \delta^{\alpha\beta}B_i^\alpha)B^{\beta\,i}\,. \tag{B.13}$$

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